Connaître

1 a) F b) F c) V d) F e) V f) V g) V h) F i) V j) F k) F

2 a) Exemple : $A(x) = 2 + 2x + 2x^2 + 2x^3$
A(x), polynôme réduit de degré 3, doit posséder 4 termes pour être complet.

b) Exemple : $B(x) = 2x^4 + 2x^2 + 2x + 2$
B(x), polynôme réduit de degré 4, doit posséder au maximum 4 termes pour être incomplet.

3 a) H(x) b) B(x) c) G(x) d) J(x) e) I(x) f) A(x)

4

Degré			
A(x)	B(x)	A(x) + B(x)	A(x) . B(x)
4	2	**= 4**	**= 6**
1	3	**= 3**	**= 4**
4	4	**⩽ 4**	**= 8**
a	a	**⩽ a**	**= 2a**

Degré			
A(x)	B(x)	A(x) + B(x)	A(x) . B(x)
a	a + 1	**= a + 1**	**= 2a + 1**
a	a – 2	**= a**	**= 2a – 2**
a – 2	a – 1	**= a – 1**	**= 2a – 3**
a	2a	**= 2a**	**= 3a**

5 a) $(a - b)^2 \neq (a + b)^2$
$(a - b)^2 = (b - a)^2$
$(a - b)^2 = (-a + b)^2$
$(a - b)^2 \neq (-a - b)^2$
$(a + b)^2 = (-a - b)^2$

b) $(a + b) \cdot (a - b) \neq (a - b) \cdot (a - b)$
$(a + b) \cdot (a - b) = (b + a) \cdot (a - b)$
$(a + b) \cdot (a - b) \neq (b + a) \cdot (b - a)$
$(a + b) \cdot (a - b) \neq (-a + b) \cdot (a + b)$
$(a + b) \cdot (a - b) = (-a + b) \cdot (-a - b)$

6 a) terme du degré le plus élevé du quotient : $2x^2$
d° Q(x) = 2
d° R(x) < 2

b) terme du degré le plus élevé du quotient : –3
d° Q(x) = 0
d° R(x) < 3

c) terme du degré le plus élevé du quotient : –2x
d° Q(x) = 1
d° R(x) < 4

d) terme du degré le plus élevé du quotient : $-\frac{3}{2}x^3$
d° Q(x) = 3
d° R(x) < 2

e) terme du degré le plus élevé du quotient : x
d° Q(x) = 1
d° R(x) < 1 ⇒ d° R(x) = 0

f) terme du degré le plus élevé du quotient : $3x^3$
d° Q(x) = 3
d° R(x) < 1 ⇒ d° R(x) = 0

Appliquer

1 $P = 4 \cdot (x + 4) = 4x + 16$ $A = (x + 4)^2 = x^2 + 8x + 16$

2 a) $P = (x + 4 + x) . 2 = (2x + 4) . 2 = 4x + 8$
$A = (x + 4) . x = x^2 + 4x$

b) $P = (2x + 1 + x + 1) . 2 = (3x + 2) . 2 = 6x + 4$
$A = (2x + 1) . (x + 1) = 2x^2 + 2x + x + 1 = 2x^2 + 3x + 1$

3 a) $P = 2 . (5x + 2) + 2 . (4x - 1)$
$= 10x + 4 + 8x - 2$
$= 18x + 2$

$A = (4x - 1) . (5x + 2) - (8x - 3) . ((4x - 1) - x)$
$= (4x - 1) . (5x + 2) - (8x - 3) . (4x - 1 - x)$
$= (4x - 1) . (5x + 2) - (8x - 3) . (3x - 1)$
$= 20x^2 + 8x - 5x - 2 - (24x^2 - 8x - 9x + 3)$
$= 20x^2 + 8x - 5x - 2 - 24x^2 + 8x + 9x - 3$
$= -4x^2 + 20x - 5$

b) $P = 2 . \pi . (2x - 1) = 4\pi x - 2\pi$
$A = \pi . (2x - 1)^2 = \pi . (4x^2 - 4x + 1) = 4\pi x^2 - 4\pi x + \pi$

c) $P = 2 . (3x + 2) + (4x + 2) = 6x + 4 + 4x + 2 = 10x + 6$
$$A = \frac{(4x+2).(5x-1)}{2} = \frac{20x^2 - 4x + 10x - 2}{2} = \frac{20x^2 + 6x - 2}{2} = 10x^2 + 3x - 1$$

4 Aire totale des faces : $2 . 2x . (2x - 1) + 2 . 2x . (2x + 1) + 2 . (2x + 1) . (2x - 1)$
$= 4x . (2x - 1) + 4x . (2x + 1) + 2 . (4x^2 - 1)$
$= 8x^2 - 4x + 8x^2 + 4x + 8x^2 - 2$
$= 24x^2 - 2$

Volume : $2x . (2x + 1) . (2x - 1) = 2x . (4x^2 - 1) = 8x^3 - 2x$

5 $A(2) = -3 . 2^2 + 2 - 4 = -3 . 4 + 2 - 4 = -12 + 2 - 4 = -14$
$A(3) = -3 . 3^2 + 3 - 4 = -3 . 9 + 3 - 4 = -27 + 3 - 4 = -28$
$A(-2) = -3 . (-2)^2 + (-2) - 4 = -3 . 4 - 2 - 4 = -12 - 2 - 4 = -18$
$A(1) = -3 . 1^2 + 1 - 4 = -3 . 1 + 1 - 4 = -3 + 1 - 4 = -6$
$A(-3) = -3 . (-3)^2 + (-3) - 4 = -3 . 9 - 3 - 4 = -27 - 3 - 4 = -34$
$A(-1) = -3 . (-1)^2 + (-1) - 4 = -3 . 1 - 1 - 4 = -3 - 1 - 4 = -8$
$A(0) = -3 . 0^2 + 0 - 4 = -3 . 0 + 0 - 4 = 0 + 0 - 4 = -4$
$$A\left(\frac{1}{2}\right) = -3 . \left(\frac{1}{2}\right)^2 + \frac{1}{2} - 4 = -3 . \frac{1}{4} + \frac{1}{2} - 4 = -\frac{3}{4} + \frac{1}{2} - 4 = \frac{-3 + 2 - 16}{4} = \frac{-17}{4}$$
$$A\left(\frac{2}{3}\right) = -3 . \left(\frac{2}{3}\right)^2 + \frac{2}{3} - 4 = -3 . \frac{4}{9} + \frac{2}{3} - 4 = -\frac{4}{3} + \frac{2}{3} - 4 = \frac{-4 + 2 - 12}{3} = \frac{-14}{3}$$
$$A\left(\frac{-1}{10}\right) = -3 . \left(\frac{-1}{10}\right)^2 + \frac{-1}{10} - 4 = -3 . \frac{1}{100} + \frac{-1}{10} - 4 = -\frac{3}{100} + \frac{-1}{10} - 4 = \frac{-3 - 10 - 400}{100} = \frac{-413}{100}$$

$B(2) = 2 . 2^3 - 2 + 1 = 2 . 8 - 2 + 1 = 16 - 2 + 1 = 15$
$B(3) = 2 . 3^3 - 3 + 1 = 2 . 27 - 3 + 1 = 54 - 3 + 1 = 52$
$B(-2) = 2 . (-2)^3 - (-2) + 1 = 2 . (-8) + 2 + 1 = -16 + 2 + 1 = -13$
$B(1) = 2 . 1^3 - 1 + 1 = 2 . 1 - 1 + 1 = 2 - 1 + 1 = 2$
$B(-3) = 2 . (-3)^3 - (-3) + 1 = 2 . (-27) + 3 + 1 = -54 + 3 + 1 = -50$
$B(-1) = 2 . (-1)^3 - (-1) + 1 = 2 . (-1) + 1 + 1 = -2 + 1 + 1 = 0$
$B(0) = 2 . 0^3 - 0 + 1 = 2 . 0 - 0 + 1 = 0 - 0 + 1 = 1$
$$B\left(\frac{1}{2}\right) = 2 . \left(\frac{1}{2}\right)^3 - \frac{1}{2} + 1 = 2 . \frac{1}{8} - \frac{1}{2} + 1 = \frac{1}{4} - \frac{1}{2} + 1 = \frac{1 - 2 + 4}{4} = \frac{3}{4}$$
$$B\left(\frac{2}{3}\right) = 2 . \left(\frac{2}{3}\right)^3 - \frac{2}{3} + 1 = 2 . \frac{8}{27} - \frac{2}{3} + 1 = \frac{16}{27} - \frac{2}{3} + 1 = \frac{16 - 18 + 27}{27} = \frac{25}{27}$$
$$B\left(\frac{-1}{10}\right) = 2 . \left(\frac{-1}{10}\right)^3 - \frac{-1}{10} + 1 = 2 . \frac{-1}{1000} + \frac{1}{10} + 1 = \frac{-1}{500} + \frac{1}{10} + 1 = \frac{-1 + 50 + 500}{500} = \frac{549}{500}$$

$C(2) = 2^3 + 5 \,.\, 2^2 - 4 \,.\, 2 + 2 = 8 + 5 \,.\, 4 - 4 \,.\, 2 + 2 = 8 + 20 - 8 + 2 = 22$
$C(3) = 3^3 + 5 \,.\, 3^2 - 4 \,.\, 3 + 2 = 27 + 5 \,.\, 9 - 4 \,.\, 3 + 2 = 27 + 45 - 12 + 2 = 62$
$C(-2) = (-2)^3 + 5 \,.\, (-2)^2 - 4 \,.\, (-2) + 2 = -8 + 5 \,.\, 4 - 4 \,.\, (-2) + 2 = -8 + 20 + 8 + 2 = 22$
$C(1) = 1^3 + 5 \,.\, 1^2 - 4 \,.\, 1 + 2 = 1 + 5 \,.\, 1 - 4 \,.\, 1 + 2 = 1 + 5 - 4 + 2 = 4$
$C(-3) = (-3)^3 + 5 \,.\, (-3)^2 - 4 \,.\, (-3) + 2 = -27 + 5 \,.\, 9 - 4 \,.\, (-3) + 2 = -27 + 45 + 12 + 2 = 32$
$C(-1) = (-1)^3 + 5 \,.\, (-1)^2 - 4 \,.\, (-1) + 2 = -1 + 5 \,.\, 1 - 4 \,.\, (-1) + 2 = -1 + 5 + 4 + 2 = 10$
$C(0) = 0^3 + 5 \,.\, 0^2 - 4 \,.\, 0 + 2 = 0 + 5 \,.\, 0 - 4 \,.\, 0 + 2 = 0 + 0 - 0 + 2 = 2$

$$C\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 5 \,.\, \left(\frac{1}{2}\right)^2 - 4 \,.\, \frac{1}{2} + 2 = \frac{1}{8} + 5 \,.\, \frac{1}{4} - 4 \,.\, \frac{1}{2} + 2 = \frac{1}{8} + \frac{5}{4} - \cancel{2} + \cancel{2}$$
$$= \frac{1+10}{8} = \frac{11}{8}$$

$$C\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 + 5 \,.\, \left(\frac{2}{3}\right)^2 - 4 \,.\, \frac{2}{3} + 2 = \frac{8}{27} + 5 \,.\, \frac{4}{9} - 4 \,.\, \frac{2}{3} + 2 = \frac{8}{27} + \frac{20}{9} - \frac{8}{3} + 2$$
$$= \frac{8 + 60 - 72 + 54}{27} = \frac{50}{27}$$

$$C\left(\frac{-1}{10}\right) = \left(\frac{-1}{10}\right)^3 + 5 \,.\, \left(\frac{-1}{10}\right)^2 - 4 \,.\, \frac{-1}{10} + 2 = \frac{-1}{1000} + 5 \,.\, \frac{1}{100} + 4 \,.\, \frac{1}{10} + 2$$
$$= \frac{-1}{1000} + \frac{1}{20} + \frac{2}{5} + 2$$
$$= \frac{-1 + 50 + 400 + 2000}{1000} = \frac{2449}{1000}$$

$D(2) = -2^3 + 4 \,.\, 2^2 - 2 \,.\, 2 - 1 = -8 + 4 \,.\, 4 - 2 \,.\, 2 - 1 = -8 + 16 - 4 - 1 = 3$
$D(3) = -3^3 + 4 \,.\, 3^2 - 2 \,.\, 3 - 1 = -27 + 4 \,.\, 9 - 2 \,.\, 3 - 1 = -27 + 36 - 6 - 1 = 2$
$D(-2) = -(-2)^3 + 4 \,.\, (-2)^2 - 2 \,.\, (-2) - 1 = -(-8) + 4 \,.\, 4 - 2 \,.\, (-2) - 1 = 8 + 16 + 4 - 1 = 27$
$D(1) = -1^3 + 4 \,.\, 1^2 - 2 \,.\, 1 - 1 = -1 + 4 \,.\, 1 - 2 \,.\, 1 - 1 = -1 + 4 - 2 - 1 = 0$
$D(-3) = -(-3)^3 + 4 \,.\, (-3)^2 - 2 \,.\, (-3) - 1 = -(-27) + 4 \,.\, 9 - 2 \,.\, (-3) - 1 = 27 + 36 + 6 - 1 = 68$
$D(-1) = -(-1)^3 + 4 \,.\, (-1)^2 - 2 \,.\, (-1) - 1 = -(-1) + 4 \,.\, 1 - 2 \,.\, (-1) - 1 = 1 + 4 + 2 - 1 = 6$
$D(0) = -0^3 + 4 \,.\, 0^2 - 2 \,.\, 0 - 1 = 0 + 4 \,.\, 0 - 2 \,.\, 0 - 1 = 0 + 0 - 0 - 1 = -1$

$$D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^3 + 4 \,.\, \left(\frac{1}{2}\right)^2 - 2 \,.\, \frac{1}{2} - 1 = -\frac{1}{8} + 4 \,.\, \frac{1}{4} - 2 \,.\, \frac{1}{2} - 1 = -\frac{1}{8} + \cancel{1} - \cancel{1} - 1$$
$$= \frac{-1-8}{8} = \frac{-9}{8}$$

$$D\left(\frac{2}{3}\right) = -\left(\frac{2}{3}\right)^3 + 4 \,.\, \left(\frac{2}{3}\right)^2 - 2 \,.\, \frac{2}{3} - 1 = -\frac{8}{27} + 4 \,.\, \frac{4}{9} - 2 \,.\, \frac{2}{3} - 1 = -\frac{8}{27} + \frac{16}{9} - \frac{4}{3} - 1$$
$$= \frac{-8 + 48 - 36 - 27}{27} = \frac{-23}{27}$$

$$D\left(\frac{-1}{10}\right) = -\left(\frac{-1}{10}\right)^3 + 4 \,.\, \left(\frac{-1}{10}\right)^2 - 2 \,.\, \frac{-1}{10} - 1 = -\frac{-1}{1000} + 4 \,.\, \frac{1}{100} - 2 \,.\, \frac{-1}{10} - 1$$
$$= \frac{1}{1000} + \frac{1}{25} + \frac{1}{5} - 1$$
$$= \frac{1 + 40 + 200 - 1000}{1000} = \frac{-759}{1000}$$

6 a)
$A(0) = 0$
$3 \,.\, 0^2 + 2 \,.\, 0 + a = 0$
$3 \,.\, 0 + 2 \,.\, 0 + a = 0$
$0 + 0 + a = 0$
$a = 0$

b)
$B(-1) = -5$
$2 \,.\, (-1)^2 - a \,.\, (-1) + 3 = -5$
$2 \,.\, 1 - a \,.\, (-1) + 3 = -5$
$2 + a + 3 = -5$
$a + 5 = -5$
$a = -5 - 5$
$a = -10$

c)
$$C(2) = 1$$
$$a \cdot 2^2 + 5 \cdot 2 + 3 = 1$$
$$a \cdot 4 + 5 \cdot 2 + 3 = 1$$
$$4a + 10 + 3 = 1$$
$$4a + 13 = 1$$
$$4a = 1 - 13$$
$$4a = -12$$
$$a = -3$$

d)
$$D(-2) = -1$$
$$2 \cdot (-2)^3 + a \cdot (-2) + 1 = -1$$
$$2 \cdot (-8) + a \cdot (-2) + 1 = -1$$
$$-16 - 2a + 1 = -1$$
$$-2a - 15 = -1$$
$$-2a = -1 + 15$$
$$-2a = 14$$
$$a = -7$$

7 $A(x) = x^3 - 12x^2 - 3x - 9$
A(x) est un polynôme de degré 3 et complet.

$B(x) = -4x + 6$
B(x) est un polynôme de degré 1 et complet.

$C(x) = 2x^4 - x^3 + 2x + 1$
C(x) est un polynôme de degré 4 et incomplet (il manque le terme de degré 2).

$D(x) = -3x^5 - 5x^4 + x^3 - 2x^2 - 6x - 1$
D(x) est un polynôme de degré 5 et complet.

$E(x) = -2x^4 + 6x^2 - x$
E(x) est un polynôme de degré 4 et incomplet (il manque les termes de degré 3 et de degré 0).

$F(x) = -x^3 - 4x^2 + 6x + 1$
F(x) est un polynôme de degré 3 et complet.

8 $A(x) = -x^3 - x$

$B(x) = -2ax^2 + 2x + 2$

$C(x) = (a + b)x^3 + x^2 + (a - b)x - 4$

$D(x) = (1 - a)x^3 + (b - a)x^2 + (-2a + b)x$

$E(x) = x^2 - \sqrt{5}\,x + 3\sqrt{5}$

$F(x) = -4x^3 + x^2$

$G(x) = -\sqrt{3}x^3 + x^2 + 2x - 1$

$H(x) = 2x^2 + (\sqrt{3} - \sqrt{5})x - 4$

9 a)

Étape 1	Étape 2	Étape 3
$3 = a - 3$ $6 = a$	$-7 = c$	$2b - 1 = b$ $b = 1$

b)

Étape 1	Étape 2	Étape 3
$-5 = 2a + 1$ $-5 - 1 = 2a$ $-6 = 2a$ $-3 = a$	$-4 = a + b$ $-4 = -3 + b$ $-4 + 3 = b$ $-1 = b$	$c - 2b = 3$ $c - 2 \cdot (-1) = 3$ $c + 2 = 3$ $c = 3 - 2$ $c = 1$

c)

Étape 1	Étape 2	Étape 3
$a + b = a - b$ $a + b - a + b = 0$ $2b = 0$ $b = 0$	$0 = a - 2$ $2 = a$	$c - 3a = -9$ $c - 3 \cdot 2 = -9$ $c - 6 = -9$ $c = -3$

10 a)

A(x) =	x^3		$+ 2x$	$- 1$
B(x) =		x^2	$- 2x$	$+ 3$
A(x) + B(x) =	x^3	$+ x^2$	$(+ 0x)$	$+ 2$

A(x) =	x^3		$+ 2x$	$- 1$
−B(x) =		$- x^2$	$+ 2x$	$- 3$
A(x) − B(x) =	x^3	$- x^2$	$+ 4x$	$- 4$

$B(x) - A(x) = -(-B(x) + A(x)) = -(A(x) - B(x)) = -(x^3 - x^2 + 4x - 4) = -x^3 + x^2 - 4x + 4$

b)

$A(x) =$	x^3		$+\,2x$	$-\,1$
$B(x) =$		x^2	$-\,2x$	$+\,3$
$C(x) =$		$-\,3x^2$	$+\;x$	$-\,2$
$A(x) + B(x) + C(x) =$	x^3	$-\,2x^2$	$+\;x$	$(+\,0)$

$A(x) =$	x^3		$+\,2x$	$-\,1$
$-B(x) =$		$-\;x^2$	$+\,2x$	$-\,3$
$C(x) =$		$-\,3x^2$	$+\;x$	$-\,2$
$A(x) - B(x) + C(x) =$	x^3	$-\,4x^2$	$+\,5x$	$-\,6$

$-A(x) + B(x) - C(x) = -(A(x) - B(x) + C(x)) = -(x^3 - 4x^2 + 5x - 6) = -x^3 + 4x^2 - 5x + 6$

c)

$D(x) =$		x^2	$+\frac{1}{2}x$	$-\,3$
$E(x) =$	$-\frac{2}{3}x^3$	$-\frac{x^2}{4}$	$+\,4x$	$-\,1$
$D(x) + E(x) =$	$-\frac{2}{3}x^3$	$+\frac{3}{4}x^2$	$+\frac{9}{2}x$	$-\,4$

$E(x) =$		$-\frac{2}{3}x^3$	$-\frac{x^2}{4}$	$+\,4x$	$-\,1$
$-F(x) =$	$-\,x^4$	$-\frac{1}{4}x^3$	$-\frac{5}{2}x^2$		$+\,1$
$E(x) - F(x) =$	$-\,x^4$	$-\frac{11}{12}x^3$	$-\frac{11}{4}x^2$	$+\,4x$	$(+\,0)$

$D(x) - E(x) + F(x) = D(x) - (E(x) - F(x))$

$D(x) =$			x^2	$+\frac{1}{2}x$	$-\,3$
$-(E(x) - F(x)) =$	x^4	$+\frac{11}{12}x^3$	$+\frac{11}{4}x^2$	$-\,4x$	
$D(x) - E(x) + F(x) =$	x^4	$+\frac{11}{12}x^3$	$+\frac{15}{4}x^2$	$-\frac{7}{2}x$	$-\,3$

11 a) $(x - 1) \,.\, (x + 5) = x^2 + 5x - x - 5 = x^2 + 4x - 5$
$(x - 3) \,.\, (x - 1) = x^2 - x - 3x + 3 = x^2 - 4x + 3$
$(3x + 1) \,.\, (x - 5) = 3x^2 - 15x + x - 5 = 3x^2 - 14x - 5$

b) $(-x^4 + 1) \,.\, (2 - x^2) = -2x^4 + x^6 + 2 - x^2 = x^6 - 2x^4 - x^2 + 2$
$(3x^3 + 1) \,.\, (x^3 - 3) = 3x^6 - 9x^3 + x^3 - 3 = 3x^6 - 8x^3 - 3$
$(-7x^2 + 3) \,.\, (x^2 - 1) = -7x^4 + 7x^2 + 3x^2 - 3 = -7x^4 + 10x^2 - 3$

c) $(-2x^3 - 1) \,.\, (1 - x^4) = -2x^3 + 2x^7 - 1 + x^4 = 2x^7 + x^4 - 2x^3 - 1$
$(x^2 + 2) \,.\, (x - 3) = x^3 - 3x^2 + 2x - 6$
$(3x^4 + 1) \,.\, (2 - x^4) = 6x^4 - 3x^8 + 2 - x^4 = -3x^8 + 5x^4 + 2$

d) $(\sqrt{3}\,x - 2) \,.\, (\sqrt{3}\,x + 1) = 3x^2 + \sqrt{3}\,x - 2\sqrt{3}\,x - 2 = 3x^2 - \sqrt{3}\,x - 2$
$(2\sqrt{2}\,x + 3) \,.\, (\sqrt{2}\,x - 2) = 4x^2 - 4\sqrt{2}\,x + 3\sqrt{2}\,x - 6 = 4x^2 - \sqrt{2}\,x - 6$
$(\sqrt{5}\,x - 3\sqrt{2}\,y) \,.\, (-\sqrt{5}\,x - 2\sqrt{2}\,y) = -5x^2 - 2\sqrt{10}\,xy + 3\sqrt{10}\,xy + 12y^2 = -5x^2 + \sqrt{10}\,xy + 12y^2$

e) $(x - y) \,.\, (x + 2y) = x^2 + 2xy - xy - 2y^2 = x^2 + xy - 2y^2$
$(5x + y) \,.\, (x - 3y) = 5x^2 - 15xy + xy - 3y^2 = 5x^2 - 14xy - 3y^2$
$(-2x + y) \,.\, (-x + 3y) = 2x^2 - 6xy - xy + 3y^2 = 2x^2 - 7xy + 3y^2$

f) $(x^3 - y) \cdot (2x^3 + 3y) = 2x^6 + 3x^3y - 2x^3y - 3y^2 = 2x^6 + x^3y - 3y^2$
$(-x^5 + y^3) \cdot (y^3 - 2x^5) = -x^5y^3 + 2x^{10} + y^6 - 2x^5y^3 = 2x^{10} - 3x^5y^3 + y^6$
$(x^3 + 2y^2) \cdot (x^3 - y^2) = x^6 - x^3y^2 + 2x^3y^2 - 2y^4 = x^6 + x^3y^2 - 2y^4$

12 a)

$$\begin{array}{lcccccc}
B(x) = & & & -2x^3 & +x^2 & -x & +3 \\
A(x) = & & & & 3x^2 & & -1 \\
\hline
 & & & +2x^3 & -x^2 & +x & -3 \\
 & -6x^5 & +3x^4 & -3x^3 & +9x^2 & & \\
\hline
B(x) \cdot A(x) = & -6x^5 & +3x^4 & -x^3 & +8x^2 & +x & -3
\end{array}$$

$$\begin{aligned}2B(x) \cdot 3A(x) &= 6 \cdot (B(x) \cdot A(x)) = 6 \cdot (-6x^5 + 3x^4 - x^3 + 8x^2 + x - 3) \\ &= -36x^5 + 18x^4 - 6x^3 + 48x^2 + 6x - 18\end{aligned}$$

$$\begin{aligned}5B(x) \cdot 2A(x) &= 10 \cdot (B(x) \cdot A(x)) = 10 \cdot (-6x^5 + 3x^4 - x^3 + 8x^2 + x - 3) \\ &= -60x^5 + 30x^4 - 10x^3 + 80x^2 + 10x - 30\end{aligned}$$

b)

$$\begin{array}{lccccccc}
B(x) = & & & & -2x^3 & +x^2 & -x & +3 \\
C(x) = & & & & -x^3 & +3x^2 & & -2 \\
\hline
 & & & & +4x^3 & -2x^2 & +2x & -6 \\
 & & -6x^5 & +3x^4 & -3x^3 & +9x^2 & & \\
 & +2x^6 & -x^5 & +x^4 & -3x^3 & & & \\
\hline
B(x) \cdot C(x) = & +2x^6 & -7x^5 & +4x^4 & -2x^3 & +7x^2 & +2x & -6
\end{array}$$

$-B(x) \cdot (-C(x)) = B(x) \cdot C(x) = 2x^6 - 7x^5 + 4x^4 - 2x^3 + 7x^2 + 2x - 6$

$$\begin{aligned}-B(x) \cdot C(x) &= -(B(x) \cdot C(x)) = -(2x^6 - 7x^5 + 4x^4 - 2x^3 + 7x^2 + 2x - 6) \\ &= -2x^6 + 7x^5 - 4x^4 + 2x^3 - 7x^2 - 2x + 6\end{aligned}$$

c)

$$\begin{array}{lccccccc}
E(x) = & & & & -\frac{1}{2}x^3 & +x^2 & -2x & -1 \\
F(x) = & & & & x^3 & & -\frac{2}{3}x & -1 \\
\hline
 & & & & +\frac{1}{2}x^3 & -x^2 & +2x & +1 \\
 & & & +\frac{1}{3}x^4 & -\frac{2}{3}x^3 & +\frac{4}{3}x^2 & +\frac{2}{3}x & \\
 & -\frac{1}{2}x^6 & +x^5 & -2x^4 & -1x^3 & & & \\
\hline
E(x) \cdot F(x) = & -\frac{1}{2}x^6 & +x^5 & -\frac{5}{3}x^4 & -\frac{7}{6}x^3 & +\frac{1}{3}x^2 & +\frac{8}{3}x & +1
\end{array}$$

$$\begin{aligned}E(x) \cdot 2F(x) &= 2 \cdot (E(x) \cdot F(x)) = 2 \cdot \left(-\frac{1}{2}x^6 + x^5 - \frac{5}{3}x^4 - \frac{7}{6}x^3 + \frac{1}{3}x^2 + \frac{8}{3}x + 1\right) \\ &= -x^6 + 2x^5 - \frac{10}{3}x^4 - \frac{7}{3}x^3 + \frac{2}{3}x^2 + \frac{16}{3}x + 2\end{aligned}$$

$$\begin{aligned}-2 \cdot E(x) \cdot F(x) &= -2 \cdot (E(x) \cdot F(x)) = = -2 \cdot \left(-\frac{1}{2}x^6 + x^5 - \frac{5}{3}x^4 - \frac{7}{6}x^3 + \frac{1}{3}x^2 + \frac{8}{3}x + 1\right) \\ &= x^6 - 2x^5 + \frac{10}{3}x^4 + \frac{7}{3}x^3 - \frac{2}{3}x^2 - \frac{16}{3}x - 2\end{aligned}$$

d)

$D(x) =$				x^3	$-3x^2$	$-2x$	$+1$
$C(x) =$				$-x^3$	$+3x^2$		-2
				$-2x^3$	$+6x^2$	$+4x$	-2
		$+3x^5$	$-9x^4$	$-6x^3$	$+3x^2$		
	$-x^6$	$+3x^5$	$+2x^4$	$-x^3$			
$D(x) . C(x) =$	$-x^6$	$+6x^5$	$-7x^4$	$-9x^3$	$+9x^2$	$+4x$	-2

$D(x) . C(x) =$			$-x^6$	$+6x^5$	$-7x^4$	$-9x^3$	$+9x^2$	$+4x$	-2
$A(x) =$							$3x^2$		-1
			$+x^6$	$-6x^5$	$+7x^4$	$+9x^3$	$-9x^2$	$-4x$	$+2$
	$-3x^8$	$+18x^7$	$-21x^6$	$-27x^5$	$+27x^4$	$+12x^3$	$-6x^2$		
$C(x) . D(x) . A(x) =$	$-3x^8$	$+18x^7$	$-20x^6$	$-33x^5$	$+34x^4$	$+21x^3$	$-15x^2$	$-4x$	$+2$

$$\begin{aligned}-10D(x) . (-C(x)) . (-A(x)) &= -10 . (D(x) . C(x) . A(x))\\ &= -10 . (-3x^8 + 18x^7 - 20x^6 - 33x^5 + 34x^4 + 21x^3 - 15x^2 - 4x + 2)\\ &= 30x^8 - 180x^7 + 200x^6 + 330x^5 - 340x^4 - 210x^3 + 150x^2 + 40x - 20\end{aligned}$$

$$\begin{aligned}D(x) . (-5C(x)) . (-2A(x)) = &= 10 . (D(x) . C(x) . A(x))\\ &= 10 . (-3x^8 + 18x^7 - 20x^6 - 33x^5 + 34x^4 + 21x^3 - 15x^2 - 4x + 2)\\ &= -30x^8 + 180x^7 - 200x^6 - 330x^5 + 340x^4 + 210x^3 - 150x^2 - 40x + 20\end{aligned}$$

13

a) $x^2 - 9$
$9x^2 - 1$
$25 - 4a^2$
$49 - 4x^2$

b) $x^2 + 8x + 16$
$49 + 28x + 4x^2$
$25a^2 - 30a + 9$
$9a^2 - 30a + 25$

c) $x^6 - 4$
$9a^8 - 4$
$25 - 9x^4$
$1 - 4x^6$

d) $x^6 + 8x^3 + 16$
$25a^2 - 10a^3 + a^4$
$4x^6 - 12x^4 + 9x^2$
$9a^6 + 12a^5 + 4a^4$

e) $5 - 9x^2$
$x^2 - 12$
$x^4 - 2$
$36x^4 - 54$

f) $9x^2 + 6\sqrt{2}x + 2$
$75 - 10\sqrt{3} + x^2$
$96 + 8\sqrt{12}x^2 + 2x^4 = 96 + 16\sqrt{3}x^2 + 2x^4$
$16x^4 - 16\sqrt{3}x^2 + 12$

g) $9 - x^2y^2$
$x^2 - 9y^2$
$x^4 - 16y^2$
$x^2y^4 - 9$

h) $4x^2 + 12xy + 9y^2$
$x^2 - 4xy + 4y^2$
$49x^4 - 14x^2y + y^2$
$25x^4y^2 + 30x^3y^4 + 9x^2y^6$

i) $1 - \frac{9}{4}x^6$
$16x^4 - \frac{1}{9}y^2$
$\frac{4}{9}x^4 - 4x^2 + 9$
$\frac{x^8}{16} + \frac{1}{3}x^4 + \frac{4}{9}$

g) $\frac{x^2}{9} - \frac{9y^2}{4}$
$\frac{x^6}{4} - \frac{9y^4}{25}$
$\frac{x^2y^2}{36} + \frac{xy}{9} + \frac{1}{9}$
$\frac{9}{4}x^2 - 3xy^2 + y^4$

14

$$\begin{aligned}P(a + 1) - P(a - 1) &= 0\\ (3 . (a + 1)^2 - 2 . (a + 1) + 4) - (3 . (a - 1)^2 - 2 . (a - 1) + 4) &= 0\\ (3 . (a^2 + 2a + 1) - 2 . (a + 1) + 4) - (3 . (a^2 - 2a + 1) - 2 . (a - 1) + 4) &= 0\\ (3a^2 + 6a + 3 - 2a - 2 + 4) - (3a^2 - 6a + 3 - 2a + 2 + 4) &= 0\\ (3a^2 + 4a + 5) - (3a^2 - 8a + 9) &= 0\\ 3a^2 + 4a + 5 - 3a^2 + 8a - 9 &= 0\\ 12a - 4 &= 0\\ 12a &= 4\\ a &= \frac{4}{12}\\ a &= \frac{1}{3}\end{aligned}$$

Chapitre 4

15 a) $9x^2 + 12x + 4 - (4x^2 - 1) = 9x^2 + 12x + 4 - 4x^2 + 1 = 5x^2 + 12x + 5$
$-15x + 9x^2 - 3x^2 + 2x = 6x^2 - 13x$
$6x^2 + 10x + x^2 - 10x + 25 = 7x^2 + 25$
$9x^2 - 4 - (4x^2 - 12x + 9) = 9x^2 - 4 - 4x^2 + 12x - 9 = 5x^2 + 12x - 13$
$25x^2 - 9 - 15x - 3x^2 = 22x^2 - 15x - 9$

b) $3x\,.\,(x^2 - 6x + 9) + x^2 - 9 = 3x^3 - 18x^2 + 27x + x^2 - 9 = 3x^3 - 17x^2 + 27x - 9$
$(4x^2 - 3)^2 - (4x - 3)^2 = 16x^4 - 24x^2 + 9 - (16x^2 - 24x + 9) = 16x^4 - 24x^2 + 9 - 16x^2 + 24x - 9$
$= 16x^4 - 40x^2 + 24x$
$-2x\,.\,(4x^2 - 4x + 1) - 8x - 12x^2 = -8x^3 + 8x^2 - 2x - 8x - 12x^2 = -8x^3 - 4x^2 - 10x$
$-x\,.\,(9x^2 - 6x + 1) + 2\,.\,(x^4 - 6x^2 + 9) = -9x^3 + 6x^2 - x + 2x^4 - 12x^2 + 18 = 2x^4 - 9x^3 - 6x^2 - x + 18$
$16x^4 - 8x^2 + 1 - (9x^2 - 1) = 16x^4 - 8x^2 + 1 - 9x^2 + 1 = 16x^4 - 17x^2 + 2$

c) $x^6 - 4x^2 - 5x^4 + 10x^2 = x^6 - 5x^4 + 6x^2$
$x^2 - 49 - (9x^2 - 42x + 49) = x^2 - 49 - 9x^2 + 42x - 49 = -8x^2 + 42x - 98$
$9 - 4x^2 - 3 + 2x = -4x^2 + 2x + 6$
$4x^2 - 6x + 9 - 4x^2 = -6x + 9$
$-(4x^2 - 4x + 1) - (1 - 4x^2) = -4x^2 + 4x - 1 - 1 + 4x^2 = 4x - 2$

d) $3x\,.\,(x^2 - 9) = 3x^3 - 27x$
$(2x - 5)\,.\,(1 - 2x)^2 = (2x - 5)\,.\,(1 - 4x + 4x^2) = 2x - 8x^2 + 8x^3 - 5 + 20x - 20x^2$
$= 8x^3 - 28x^2 + 22x - 5$
$(x^2 - 4x + 4)\,.\,(2x + 1) = 2x^3 + x^2 - 8x^2 - 4x + 8x + 4 = 2x^3 - 7x^2 + 4x + 4$
$-2x\,.\,(9x^2 - 4)\,.\,(9x^2 - 4) = -2x\,.\,(9x^2 - 4)^2 = -2x\,.\,(81x^4 - 72x^2 + 16) = -162x^5 + 144x^3 - 32x$
$(4x^2 - 1)\,.\,(4x^2 + 1) = 16x^4 - 1$

16 a) $27x^3 + 54x^2 + 36x + 8$
$-x^3 + 15x^2 - 75x + 125$
$-64 - 144x - 108x^2 - 27x^3 = -27x^3 - 108x^2 - 144x - 64$

b) $125x^3 + 300x^2y + 240xy^2 + 64y^3$
$27x^3 - 54x^2y + 36xy^2 - 8y^3$
$8x^3 - 36x^2 + 54x - 27$

c) $27a^6 + 54a^4b + 36a^2b^2 + 8b^3$
$x^6 + 6x^4y^3 + 12x^2y^6 + 8y^9$
$-8x^6 + 36x^4y^2 - 54x^2y^4 + 27y^6$

d) $x^3 + x^2 + \frac{1}{3}x + \frac{1}{27}$

$-27x^3 - \frac{27}{2}x^2 - \frac{9}{4}x - \frac{1}{8}$

$\frac{1}{125}x^3 - \frac{9}{50}x^2y + \frac{27}{20}xy^2 - \frac{27}{8}y^3$

17 a) $(x^3 + 3x^2 - 7x + 2) : (x + 3)$

x^3	$+ 3x^2$	$- 7x$	$+ 2$	$x + 3$
$- x^3$	$- 3x^2$			$x^2 - 7$
		$- 7x$	$+ 2$	
		$+ 7x$	$+ 21$	
			$+ 23$	

$x^3 + 3x^2 - 7x + 2 = (x + 3)\,.\,(x^2 - 7) + 23$

Chapitre 4

$(x^3 + x^2 + x + 1) : (x^2 - 1)$

$$\begin{array}{rrrr|l}
x^3 & +x^2 & +x & +1 & x^2 - 1 \\ \cline{5-5}
-x^3 & & +x & & x + 1 \\ \cline{1-4}
 & x^2 & +2x & +1 & \\
 & -x^2 & & +1 & \\ \cline{1-4}
 & & 2x & +2 &
\end{array}$$

$x^3 + x^2 + x + 1 = (x^2 - 1) . (x + 1) + 2x + 2$

$(2x^3 - 9x^2 + 13x - 6) : (2x - 3)$

$$\begin{array}{rrrr|l}
2x^3 & -9x^2 & +13x & -6 & 2x - 3 \\ \cline{5-5}
-2x^3 & +3x^2 & & & x^2 - 3x + 2 \\ \cline{1-4}
 & -6x^2 & +13x & -6 & \\
 & +6x^2 & -\ 9x & & \\ \cline{1-4}
 & & 4x & -6 & \\
 & & -4x & +6 & \\ \cline{1-4}
 & & & 0 &
\end{array}$$

$2x^3 - 9x^2 + 13x - 6 = (2x - 3) . (x^2 - 3x + 2)$

$(6x^3 + 17x^2 - x - 4) : (3x + 1)$

$$\begin{array}{rrrr|l}
6x^3 & +17x^2 & -x & -4 & 3x + 1 \\ \cline{5-5}
-6x^3 & -\ 2x^2 & & & 2x^2 + 5x - 2 \\ \cline{1-4}
 & +15x^2 & -\ x & -4 & \\
 & -15x^2 & -5x & & \\ \cline{1-4}
 & & -6x & -4 & \\
 & & +6x & +2 & \\ \cline{1-4}
 & & & -2 &
\end{array}$$

$6x^3 + 17x^2 - x - 4 = (3x + 1) . (2x^2 + 5x - 2) - 2$

b) $(3x^4 - 5x^2 + 2) : (3x^2 - 2)$

$$\begin{array}{rrr|l}
3x^4 & -5x^2 & +2 & 3x^2 - 2 \\ \cline{4-4}
-3x^4 & +2x^2 & & x^2 - 1 \\ \cline{1-3}
 & -3x^2 & +2 & \\
 & +3x^2 & -2 & \\ \cline{1-3}
 & & 0 &
\end{array}$$

$3x^4 - 5x^2 + 2 = (3x^2 - 2) . (x^2 - 1)$

$(4x^5 - 5x^4 + 1) : (x - 1)$

$4x^5$	$-5x^4$				$+1$	$x-1$
$-4x^5$	$+4x^4$					$4x^4 - x^3 - x^2 - x - 1$
	$-x^4$				$+1$	
	$+x^4$	$-x^3$				
		$-x^3$			$+1$	
		$+x^3$	$-x^2$			
			$-x^2$		$+1$	
			$+x^2$	$-x$		
				$-x$	$+1$	
				$+x$	-1	
					0	

$4x^5 - 5x^4 + 1 = (x - 1) \,.\, (4x^4 - x^3 - x^2 - x - 1)$

$(x^5 + 1) : (x^2 - x + 1)$

x^5					$+1$	$x^2 - x + 1$
$-x^5$	$+x^4$	$-x^3$				$x^3 + x^2 - 1$
	x^4	$-x^3$			$+1$	
	$-x^4$	$+x^3$	$-x^2$			
			$-x^2$		$+1$	
			$+x^2$	$-x$	$+1$	
				$-x$	$+2$	

$x^5 + 1 = (x^2 - x + 1) \,.\, (x^3 + x^2 - 1) - x + 2$

$(x^4 + 3x^3 - x + 1) : (x^2 - 4x + 1)$

x^4	$+3x^3$		$-x$	$+1$	$x^2 - 4x + 1$
$-x^4$	$+4x^3$	$-x^2$			$x^2 + 7x + 27$
	$+7x^3$	$-x^2$	$-x$	$+1$	
	$-7x^3$	$+28x^2$	$-7x$		
		$+27x^2$	$-8x$	$+1$	
		$-27x^2$	$+108x$	-27	
			$100x$	-26	

$x^4 + 3x^3 - x + 1 = (x^2 - 4x + 1) \,.\, (x^2 + 7x + 27) + 100x - 26$

18 a) $(2x^2 - 5x - 3) : (x - 3)$

	2	–5	–3
		+	+
a = 3		6	3
	2	1	0

$Q(x) = 2x + 1$ et $r = 0$

b) $(x^4 + x^3 - 2x^2 + x + 3) : (x + 1)$

	1	1	–2	1	3
		+	+	+	+
a = –1		–1	0	2	–3
	1	0	–2	3	0

$Q(x) = x^3 - 2x + 3$ et $r = 0$

$(3x^3 - 7x^2 + 5x - 10) : (x - 2)$

	3	–7	5	–10
		+	+	+
a = 2		6	–2	6
	3	–1	3	–4

$Q(x) = 3x^2 - x + 3$ et $r = -4$

$(x^3 - 2x^2 + x - 6) : (x + 2)$

	1	–2	1	–6
		+	+	+
a = –2		–2	8	–18
	1	–4	9	–24

$Q(x) = x^2 - 4x + 9$ et $r = -24$

$(x^4 + x^3 - 2x^2 + 3x - 3) : (x - 1)$

	1	1	–2	3	–3
		+	+	+	+
a = 1		1	2	0	3
	1	2	0	3	0

$Q(x) = x^3 + 2x^2 + 3$ et $r = 0$

$(2x^4 - 5x^3 + 6x^2 - 7x + 4) : (x - 2)$

	2	–5	6	–7	4
		+	+	+	+
a = 2		4	–2	8	2
	2	–1	4	1	6

$Q(x) = 2x^3 - x^2 + 4x + 1$ et $r = 6$

c) $(5x^2 - 1) : (x + 1)$

	5	0	–1
		+	+
a = –1		–5	5
	5	–5	4

$Q(x) = 5x - 5$ et $r = 4$

$(x^3 + 27) : (x + 3)$

	1	0	0	27
		+	+	+
a = –3		–3	9	–27
	1	–3	9	0

$Q(x) = x^2 - 3x + 9$ et $r = 0$

$(2x^3 - 2x - 4) : (x - 2)$

	2	0	–2	–4
		+	+	+
a = 2		4	8	12
	2	4	6	8

$Q(x) = 2x^2 + 4x + 6$ et $r = 8$

d) $(5x^4 - 3x^2 + 2) : (x - 3)$

	5	0	–3	0	2
		+	+	+	+
a = 3		15	45	126	378
	5	15	42	126	380

$Q(x) = 5x^3 + 15x^2 + 42x + 126$ et $r = 380$

$(x^4 + 3x^3 + 3x - 2) : (x + 1)$

	1	3	0	3	–2
		+	+	+	+
a = –1		–1	–2	2	–5
	1	2	–2	5	–7

$Q(x) = x^3 + 2x^2 - 2x + 5$ et $r = -7$

$(x^5 - x^3 + 2x + 3) : (x - 1)$

	1	0	–1	0	2	3
		+	+	+	+	+
a = 1		1	1	0	0	2
	1	1	0	0	2	5

$Q(x) = x^4 + x^3 + 2$ et $r = 5$

19 $A(x) = D(x) . Q(x)$

$A(x) = (2x + 3) . (2x - 1) = 4x^2 - 2x + 6x - 3 = 4x^2 + 4x - 3$

20 $A(x) = D(x) . Q(x)$

En utilisant la méthode des coefficients indéterminés, on a

$x^3 - x^2 - 9x + a = (x - 3) . (mx^2 + nx + p)$
$x^3 - x^2 - 9x + a = mx^3 + nx^2 + px - 3mx^2 - 3nx - 3p$
$x^3 - x^2 - 9x + a = mx^3 + (n - 3m)x^2 + (p - 3n)x - 3p$

$1 = m$	$-1 = n - 3m$	$-9 = p - 3n$	$a = -3p$
	$-1 = n - 3 . 1$	$-9 = p - 3 . 2$	$a = -3 . (-3)$
	$-1 = n - 3$	$-9 = p - 6$	$\mathbf{a = 9}$
	$2 = n$	$-3 = p$	

En utilisant le tableau d'Horner, on a

	1	–1	–9	a
		+	+	+
a = 3		3	6	–9
	1	2	–3	0

$a + (-9) = 0$
$\mathbf{a = 9}$

Autre méthode :
$A(3) = 0$
$3^3 - 3^2 - 9 \cdot 3 + a = 0$
$27 - 9 - 27 + a = 0$
$-9 + a = 0$
$\mathbf{a = 9}$

21 $A(x) = D(x) \cdot Q(x) + r$

$4x^3 + x + a = (2x + 1) \cdot (mx^2 + nx + p) + 3$
$4x^3 + x + a = 2mx^3 + 2nx^2 + 2px + mx^2 + nx + p + 3$
$4x^3 + x + a = 2mx^3 + (2n + m)x^2 + (2p + n)x + (p + 3)$

$4 = 2m$
$2 = m$

$0 = 2n + m$
$0 = 2n + 2$
$-2 = 2n$
$-1 = n$

$1 = 2p + n$
$1 = 2p - 1$
$2 = 2p$
$1 = p$

$a = p + 3$
$a = 1 + 3$
$\mathbf{a = 4}$

Transférer

1 $2 \cdot ((2x + 9) + (2x - 5)) = 64$
$2 \cdot (2x + 9 + 2x - 5) = 64$
$2 \cdot (4x + 4) = 64$
$8x + 8 = 64$
$8x = 56$
$x = 7$

La valeur de x doit être de 7 cm.

Vérification :
Longueur du rectangle : $2x + 9 = 2 \cdot 7 + 9 = 14 + 9 = 23$ cm
Largeur du rectangle : $2x - 5 = 2 \cdot 7 - 5 = 14 - 5 = 9$ cm
Périmètre du rectangle : $(23 + 9) \cdot 2 = 32 \cdot 2 =$ **64 cm**

2 $\frac{((x+4)+(x-2)) \cdot (x-1)}{2} = 48$

$\frac{(x+4+x-2) \cdot (x-1)}{2} = 48$

$\frac{(2x+2) \cdot (x-1)}{2} = 48$

$\frac{2x^2 - 2x + 2x - 2}{2} = 48$

$\frac{2x^2 - 2}{2} = 48$

$\frac{2 \cdot (x^2 - 1)}{2} = 48$

$x^2 - 1 = 48$

$x^2 = 49$

$x = 7$ (La valeur négative est à rejeter.)

La valeur de x doit être de 7 cm.

Vérification :

Longueur de la grande base du trapèze : $7 + 4 = 11$ cm
Longueur de la petite base du trapèze : $7 - 2 = 5$ cm
Hauteur du trapèze : $7 - 1 = 6$ cm
Aire du trapèze : $((11 + 5) \cdot 6)) : 2 = (16 \cdot 6) : 2 = 96 : 2 =$ **48 cm²**

3

$$x^2 - (x-6)^2 = 96$$
$$x^2 - (x^2 - 12x + 36) = 96$$
$$x^2 - x^2 + 12x - 36 = 96$$
$$12x - 36 = 96$$
$$12x = 132$$
$$x = 11$$

La valeur de x doit être de 11 cm.

Vérification :

Côté du grand carré : 11 cm
Aire du grand carré : $11 . 11 = 121$ cm^2
Côté du petit carré : $11 - 2 . 3 = 11 - 6 = 5$ cm
Aire du petit carré : $5 . 5 = 25$ cm^2
Aire de la partie colorée : $121 - 25 =$ **96 cm^2**

4

$$(30 - 2x) . (20 - 2x) = 4x^2$$
$$600 - 60x - 40x + 4x^2 = 4x^2$$
$$-60x - 40x + 4x^2 - 4x^2 = -600$$
$$-100x = -600$$
$$x = 6$$

La valeur de x doit être de 6 cm.

Vérification :

Côté d'un petit carré : 6 cm
Aire d'un petit carré : $6 . 6 = 36$ cm^2
Aire des 4 petits carrés : $4 . 36 =$ **144 cm^2**
Longueur du rectangle coloré : $30 - 2 . 6 = 30 - 12 = 18$ cm
Largeur du rectangle coloré : $20 - 2 . 6 = 20 - 12 = 8$ cm
Aire du rectangle coloré : $18 . 8 =$ **144 cm^2**

5 Soit x le nombre cherché

$$(x + 1)^2 = x^2 + 19$$
$$x^2 + 2x + 1 = x^2 + 19$$
$$x^2 + 2x - x^2 = 19 - 1$$
$$2x = 18$$
$$x = 9$$

Ce nombre est 9.

Vérification :

Nombre : 9
Nombre augmenté de 1 : $9 + 1 = 10$
Carré du nombre : $9^2 =$ **81**
Carré du nombre augmenté de 1 : $10^2 =$ **100** (**+19**)

6 Soit x le nombre cherché

$$(x - 10)^2 = x^2 - 320$$
$$x^2 - 20x + 100 = x^2 - 320$$
$$x^2 - 20x - x^2 = -320 - 100$$
$$-20x = -420$$
$$x = 21$$

Ce nombre est 21.

Vérification :

Nombre : 21
Nombre diminué de 10 : $21 - 10 = 11$
Carré du nombre : $21^2 =$ **441**
Carré du nombre diminué de 10 : $11^2 =$ **121** (**–320**)

7 Calculons $125^2 - 123^2$.

$$\begin{aligned}125^2 - 123^2 &= (124 + 1)^2 - (124 - 1)^2\\ &= 124^2 + 2 \,.\, 124 + 1 - 124^2 + 2 \,.\, 124 - 1\\ &= 4 \,.\, 124\\ &= 496\end{aligned}$$

On peut donc en déduire que $125^2 - 123^2 = 496 = 4 \,.\, 124$.

Calculons $163^2 - 161^2$.

$$\begin{aligned}1163^2 - 161^2 &= (162 + 1)^2 - (162 - 1)^2\\ &= 162^2 + 2 \,.\, 162 + 1 - 162^2 + 2 \,.\, 162 - 1\\ &= 4 \,.\, 162\\ &= 648\end{aligned}$$

On peut donc en déduire que $163^2 - 161^2 = 648 = 4 \,.\, 162$.

Calculons $201^2 - 199^2$.

$$\begin{aligned}201^2 - 199^2 &= (200 + 1)^2 - (200 - 1)^2\\ &= 200^2 + 2 \,.\, 200 + 1 - 200^2 + 2 \,.\, 200 - 1\\ &= 4 \,.\, 200\\ &= 800\end{aligned}$$

On peut donc en déduire que $201^2 - 199^2 = 800 = 4 \,.\, 200$.

Généralisons.
Si x est un nombre entier, on a

$$(x + 1)^2 - (x - 1)^2 = 4x$$

Effectuons $(x + 1)^2 - (x - 1)^2$

$$\begin{aligned}(x + 1)^2 - (x - 1)^2 &= (x^2 + 2x + 1) - (x^2 - 2x + 1)\\ &= x^2 + 2x + 1 - x^2 + 2x - 1\\ &= 4x\end{aligned}$$

On peut donc en déduire que $(x + 1)^2 - (x - 1)^2 = 4x$

Deux autres égalités du même type :

$16^2 - 14^2 = 4 \,.\, 15 = 60$

$251^2 - 249^2 = 4 \,.\, 250 = 1000$

8

$$\begin{aligned}(2x^2 + 2x + 1)^2 &= (2x + 1)^2 + (2x^2 + 2x)^2\\ (2x^2 + 2x + 1) \,.\, (2x^2 + 2x + 1) &= 4x^2 + 4x + 1 + 4x^4 + 8x^3 + 4x^2\\ 4x^4 + 4x^3 + 2x^2 + 4x^3 + 4x^2 + 2x + 2x^2 + 2x + 1 &= 4x^4 + 8x^3 + 8x^2 + 4x + 1\\ 4x^4 + 8x^3 + 8x^2 + 4x + 1 &= 4x^4 + 8x^3 + 8x^2 + 4x + 1\end{aligned}$$

$\Rightarrow (2x^2 + 2x + 1)^2 = (2x + 1)^2 + (2x^2 + 2x)^2$

$\Rightarrow$ Le triangle est rectangle. (Si dans un triangle, le carré de la longueur du plus grand côté est égal à la somme des carrés des longueurs des deux autres côtés, alors ce triangle est rectangle.)

Si x = 1, les dimensions du triangle rectangle sont **3**, **4** et **5**.

$2x + 1 = 2 \,.\, 1 + 1 = 2 + 1 = \mathbf{3}$

$2x^2 + 2x = 2 \,.\, 1^2 + 2 \,.\, 1 = 2 \,.\, 1 + 2 \,.\, 1 = 2 + 2 = \mathbf{4}$

$2x^2 + 2x + 1 = 2 \,.\, 1^2 + 2 \,.\, 1 + 1 = 2 \,.\, 1 + 2 \,.\, 1 + 1 = 2 + 2 + 1 = \mathbf{5}$

Si x = 2, les dimensions du triangle rectangle sont **5**, **12** et **13**.

$2x + 1 = 2 \,.\, 2 + 1 = 4 + 1 = \mathbf{5}$

$2x^2 + 2x = 2 \,.\, 2^2 + 2 \,.\, 2 = 2 \,.\, 4 + 2 \,.\, 2 = 8 + 4 = \mathbf{12}$

$2x^2 + 2x + 1 = 2 \,.\, 2^2 + 2 \,.\, 2 + 1 = 2 \,.\, 4 + 2 \,.\, 2 + 1 = 8 + 4 + 1 = \mathbf{13}$

Si x = 3, les dimensions du triangle rectangle sont **7**, **24** et **25**.

$2x + 1 = 2 \,.\, 3 + 1 = 6 + 1 = \mathbf{7}$

$2x^2 + 2x = 2 \,.\, 3^2 + 2 \,.\, 3 = 2 \,.\, 9 + 2 \,.\, 3 = 18 + 6 = \mathbf{24}$

$2x^2 + 2x + 1 = 2 \,.\, 3^2 + 2 \,.\, 3 + 1 = 2 \,.\, 9 + 2 \,.\, 3 + 1 = 18 + 6 + 1 = \mathbf{25}$

Si x = 10, les dimensions du triangle rectangle sont **21**, **220** et **221**.

$2x + 1 = 2 \,.\, 10 + 1 = \mathbf{21}$

$2x^2 + 2x = 2 \,.\, 10^2 + 2 \,.\, 10 = 2 \,.\, 100 + 2 \,.\, 10 = 200 + 20 = \mathbf{220}$

$2x^2 + 2x + 1 = 2 \,.\, 10^2 + 2 \,.\, 10 + 1 = 2 \,.\, 100 + 2 \,.\, 10 + 1 = 200 + 20 + 1 = \mathbf{221}$

9 Soient x et x + 1 deux entiers consécutifs

$(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$
$2x + 1$ est un nombre impair.

10 Soient x – 1, x et x + 1 trois naturels consécutifs

$$\begin{aligned}(x-1) \cdot x \cdot (x+1) + x &= x \cdot (x-1) \cdot (x+1) + x \\ &= x \cdot (x^2 - 1) + x \\ &= x^3 - x + x \\ &= x^3\end{aligned}$$

x^3 est le cube de x.

ou

$$\begin{aligned}x \cdot (x+1) \cdot (x+2) + x + 1 &= (x^2 + x) \cdot (x+2) + x + 1 \\ &= (x^3 + 2x^2 + x^2 + 2x) + x + 1 \\ &= x^3 + 3x^2 + 2x + x + 1 \\ &= x^3 + 3x^2 + 3x + 1 \\ &= (x+1)^3\end{aligned}$$

$(x + 1)^3$ est le cube de x + 1

11 Soit x et y les deux nombres cherchés

$2 \cdot (x^2 + y^2) = (x - y)^2 + (x + y)^2$
$2x^2 + 2y^2 = x^2 - 2xy + y^2 + x^2 + 2xy + y^2$
$2x^2 + 2y^2 = 2x^2 + 2y^2$

$\Rightarrow 2 \cdot (x^2 + y^2) = (x - y)^2 + (x + y)^2$

12 $4 \cdot x \cdot y = (x + y)^2 - (x - y)^2$
$4xy = x^2 + 2xy + y^2 - (x^2 - 2xy + y^2)$
$4xy = x^2 + 2xy + y^2 - x^2 + 2xy - y^2$
$4xy = 4xy$

$\Rightarrow 4 \cdot x \cdot y = (x + y)^2 - (x - y)^2$

Chapitre 4