

Exercices complémentaires sur les irrationnels

1) Simplification des radicaux

Il faut décomposer le radicand en un produit de deux facteurs dont l'un est un carré parfait.

Exemples : $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6 \cdot \sqrt{2} = 6\sqrt{2}$

$2\sqrt{24} = 2 \cdot \sqrt{4 \cdot 6} = 2 \cdot 2\sqrt{6} = 4\sqrt{6}$

Simplifie.

$\sqrt{12} = 2\sqrt{3}$	$\sqrt{28} = 2\sqrt{7}$	$\sqrt{32} = 4\sqrt{2}$	$\sqrt{45} = 3\sqrt{5}$
$\sqrt{8} = 2\sqrt{2}$	$\sqrt{48} = 4\sqrt{3}$	$\sqrt{18} = 3\sqrt{2}$	$\sqrt{40} = 2\sqrt{10}$
$\sqrt{20} = 2\sqrt{5}$	$\sqrt{27} = 3\sqrt{3}$	$\sqrt{24} = 2\sqrt{6}$	$\sqrt{50} = 5\sqrt{2}$
$\sqrt{54} = 3\sqrt{6}$	$\sqrt{75} = 5\sqrt{3}$	$\sqrt{72} = 6\sqrt{2}$	$\sqrt{63} = 3\sqrt{7}$
$\sqrt{90} = 3\sqrt{10}$	$\sqrt{150} = 5\sqrt{6}$	$\sqrt{200} = 10\sqrt{2}$	$\sqrt{160} = 4\sqrt{10}$

$3\sqrt{8} = 3 \cdot 2 \cdot \sqrt{2}$ $= 6\sqrt{2}$	$2\sqrt{12} = 2 \cdot 2 \cdot \sqrt{3}$ $= 4\sqrt{3}$	$5\sqrt{18} = 5 \cdot 3 \cdot \sqrt{2}$ $= 15\sqrt{2}$
$4\sqrt{27} = 4 \cdot 3\sqrt{3}$ $= 12\sqrt{3}$	$2\sqrt{125} = 2 \cdot 5\sqrt{5}$ $= 10\sqrt{5}$	$3\sqrt{98} = 3 \cdot 7\sqrt{2}$ $= 21\sqrt{2}$
$5\sqrt{32} = 5 \cdot 4\sqrt{2}$ $= 20\sqrt{2}$	$4\sqrt{45} = 4 \cdot 3\sqrt{5}$ $= 12\sqrt{5}$	$3\sqrt{48} = 3 \cdot 4\sqrt{3}$ $= 12\sqrt{3}$
$6\sqrt{50} = 6 \cdot 5\sqrt{2}$ $= 30\sqrt{2}$	$2\sqrt{72} = 2 \cdot 6\sqrt{2}$ $= 12\sqrt{2}$	$5\sqrt{80} = 5 \cdot 4\sqrt{5}$ $= 20\sqrt{5}$
$2\sqrt{90} = 2 \cdot 3\sqrt{10}$ $= 6\sqrt{10}$	$3\sqrt{28} = 3 \cdot 2\sqrt{7}$ $= 6\sqrt{7}$	$2\sqrt{200} = 2 \cdot 10\sqrt{2}$ $= 20\sqrt{2}$

2) Somme de radicaux

Exemples : $2\sqrt{6} + 5\sqrt{6} = 7\sqrt{6}$

$$\sqrt{8} + \sqrt{50} = 2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$$

$$3\sqrt{7} + 2\sqrt{3} = / \text{ Impossible de réduire car les radicands ne sont pas semblables.}$$

Réduis les sommes suivantes après avoir simplifié si nécessaire.

$$\begin{array}{lll} 5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3} & 2\sqrt{7} - 7\sqrt{2} = -5\sqrt{2} & 2\sqrt{7} + \sqrt{14} = - \\ 5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7} & \sqrt{6} + \sqrt{6} = 2\sqrt{6} & \sqrt{5} - 5\sqrt{5} = -4\sqrt{5} \\ \sqrt{2} + 2\sqrt{2} = 3\sqrt{2} & 5\sqrt{7} - 10\sqrt{7} = -5\sqrt{7} & 2\sqrt{6} + 4\sqrt{3} = - \end{array}$$

$$\begin{array}{ll} 5\sqrt{5} + 3\sqrt{3} + 2\sqrt{5} + 6\sqrt{3} = 7\sqrt{5} + 9\sqrt{3} & -\sqrt{7} + 2\sqrt{5} - 2\sqrt{5} - \sqrt{7} = -2\sqrt{7} \\ 2\sqrt{7} - 5\sqrt{2} + \sqrt{7} - 3\sqrt{2} = 3\sqrt{7} - 8\sqrt{2} & -2\sqrt{3} + 3\sqrt{5} - 3\sqrt{3} - 5\sqrt{5} = -5\sqrt{3} - 2\sqrt{5} \end{array}$$

$$\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$$

$$\sqrt{12} + 3\sqrt{2} - \sqrt{8} = 2\sqrt{3} + 3\sqrt{2} - 2\sqrt{2} = 2\sqrt{3} + \sqrt{2}$$

$$\sqrt{18} + \sqrt{75} - \sqrt{20} = 3\sqrt{2} + 5\sqrt{3} - 2\sqrt{5}$$

$$\sqrt{7} + \sqrt{28} - \sqrt{63} = \sqrt{7} + 2\sqrt{7} - 3\sqrt{7} = 0$$

$$\sqrt{50} + \sqrt{8} - 3\sqrt{2} = 5\sqrt{2} + 2\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$$

$$3\sqrt{18} - 4\sqrt{72} + 5\sqrt{32} = 3 \cdot 3\sqrt{2} - 4 \cdot 6\sqrt{2} + 5 \cdot 4\sqrt{2}$$

$$9\sqrt{2} - 24\sqrt{2} + 20\sqrt{2} = 5\sqrt{2}$$

$$3\sqrt{50} - 2\sqrt{5} - 2\sqrt{8} - \sqrt{45} = 3 \cdot 5\sqrt{2} - 2\sqrt{5} - 2 \cdot 2\sqrt{2} - 3\sqrt{5}$$

$$= 15\sqrt{2} - 2\sqrt{5} - 4\sqrt{2} - 3\sqrt{5} = 11\sqrt{2} - 5\sqrt{5}$$

$$\sqrt{48} - \sqrt{24} - \sqrt{150} + \sqrt{12} = 4\sqrt{3} - 2\sqrt{6} - 5\sqrt{6} + 2\sqrt{3}$$

$$= 6\sqrt{3} - 7\sqrt{6}$$

3) Produit de radicaux

Exemples : $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$

$$\sqrt{27} \cdot \sqrt{8} = 3\sqrt{3} \cdot 2\sqrt{2} = 6\sqrt{6}$$

$$\sqrt{7} \cdot \sqrt{7} = (\sqrt{7})^2 = 7$$

Réduis les produits suivants.

$$\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$$

$$4\sqrt{15} \cdot 2\sqrt{5} = 8 \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{5} = 40\sqrt{3}$$

$$\sqrt{6} \cdot \sqrt{7} = \sqrt{42}$$

$$\sqrt{10} \cdot 3\sqrt{2} = 3\sqrt{20} = 3 \cdot 2\sqrt{5} = 6\sqrt{5}$$

$$7\sqrt{2} \cdot 6\sqrt{5} = 42\sqrt{10}$$

$$4\sqrt{3} \cdot \sqrt{21} = 4\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{7} = 12\sqrt{7}$$

$$4\sqrt{3} \cdot \sqrt{10} = 4\sqrt{30}$$

$$2\sqrt{15} \cdot 5\sqrt{3} = 2 \cdot \sqrt{3} \cdot \sqrt{5} \cdot 5 \cdot \sqrt{3} = 30\sqrt{5}$$

$$2\sqrt{5} \cdot 5\sqrt{7} = 10\sqrt{35}$$

$$2\sqrt{7} \cdot 3\sqrt{14} = 2 \cdot \sqrt{7} \cdot 3 \cdot \sqrt{7} \cdot \sqrt{2} = 42\sqrt{2}$$

$$\sqrt{27} \cdot \sqrt{8} = 3\sqrt{3} \cdot 2\sqrt{2} = 6\sqrt{6}$$

$$2\sqrt{8} \cdot 4\sqrt{12} = 2 \cdot 2\sqrt{2} \cdot 4 \cdot 2\sqrt{3} = 32\sqrt{6}$$

$$\sqrt{28} \cdot \sqrt{75} = 2\sqrt{7} \cdot 5\sqrt{3} = 10\sqrt{21}$$

$$\sqrt{32} \cdot \sqrt{125} = 4\sqrt{2} \cdot 5\sqrt{5} = 20\sqrt{10}$$

$$3\sqrt{20} \cdot 2\sqrt{27} = 3 \cdot 2\sqrt{5} \cdot 2 \cdot 3\sqrt{3} = 36\sqrt{15}$$

$$\sqrt{72} \cdot \sqrt{63} = 6\sqrt{2} \cdot 3\sqrt{7} = 18\sqrt{14}$$

$$\sqrt{8} \cdot 3\sqrt{80} = 2\sqrt{2} \cdot 3 \cdot 4\sqrt{5} = 24\sqrt{10}$$

$$3\sqrt{48} \cdot 2\sqrt{18} = 3 \cdot 4\sqrt{3} \cdot 2 \cdot 3\sqrt{2} = 72\sqrt{6}$$

$$2\sqrt{24} \cdot \sqrt{27} = 2 \cdot 2\sqrt{6} \cdot 3\sqrt{3} = 12\sqrt{18} = 12 \cdot 3\sqrt{2} = 36\sqrt{2}$$

$$\sqrt{50} \cdot 2\sqrt{10} = 5\sqrt{2} \cdot 2 \cdot \sqrt{10} = 10\sqrt{20} = 10 \cdot 2\sqrt{5} = 20\sqrt{5}$$

$$4\sqrt{12} \cdot \sqrt{24} = 4 \cdot 2\sqrt{3} \cdot 2\sqrt{6} = 16\sqrt{18} = 16 \cdot 3\sqrt{2} = 48\sqrt{2}$$

$$2\sqrt{75} \cdot \sqrt{54} = 2 \cdot 5\sqrt{3} \cdot 3\sqrt{6} = 30\sqrt{18} = 30 \cdot 3\sqrt{2} = 90\sqrt{2}$$

$$\sqrt{6} \cdot \sqrt{6} = 6$$

$$\sqrt{3} \cdot \sqrt{27} = \sqrt{3} \cdot \sqrt{3} \cdot 3 = 3 \cdot 3 = 9$$

$$5\sqrt{2} \cdot \sqrt{2} = 5 \cdot 2 = 10$$

$$3\sqrt{5} \cdot \sqrt{20} = 3 \cdot \sqrt{5} \cdot \sqrt{5} \cdot \sqrt{4} = 6 \cdot 5 = 30$$

$$3\sqrt{5} \cdot 2\sqrt{5} = 10 \cdot 5 = 50$$

$$3\sqrt{7} \cdot \sqrt{28} = 3 \cdot \sqrt{7} \cdot 2\sqrt{7} = 6 \cdot 7 = 42$$

$$(\sqrt{3})^2 = 3$$

$$3 \cdot (\sqrt{5})^2 = 3 \cdot 5 = 15$$

$$(5\sqrt{2})^2 = 25 \cdot 2 = 50$$

$$(6\sqrt{3})^2 = 36 \cdot 3 = 108$$

$$(-2\sqrt{7})^2 = 4 \cdot 7 = 28$$

$$(2\sqrt{10})^2 = 4 \cdot 10 = 40$$

4) Quotient de radicaux

Il faut toujours rendre le dénominateur rationnel c'ad sans $\sqrt{\quad}$ au dénominateur en multipliant le numérateur et le dénominateur par la racine carrée gênante du dénominateur.

$$\text{Exemples : } \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

$$\sqrt{\frac{16}{27}} = \frac{\sqrt{16}}{\sqrt{27}} = \frac{4}{3\sqrt{3}} = \frac{4 \cdot \sqrt{3}}{3\sqrt{3} \cdot \sqrt{3}} = \frac{4\sqrt{3}}{3 \cdot 3} = \frac{4\sqrt{3}}{9}$$

Rends rationnel le dénominateur.

$$\begin{array}{l} \sqrt{\frac{16}{7}} = \frac{4 \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}} = \frac{4\sqrt{7}}{7} \quad \sqrt{\frac{11}{36}} = \frac{\sqrt{11}}{6} \quad \sqrt{\frac{125}{48}} = \frac{5\sqrt{5} \cdot \sqrt{3}}{4\sqrt{3} \cdot \sqrt{3}} = \frac{5\sqrt{15}}{12} \\ \sqrt{\frac{25}{9}} = \frac{5}{3} \quad \sqrt{\frac{27}{32}} = \frac{3\sqrt{3} \cdot \sqrt{2}}{4\sqrt{2} \cdot \sqrt{2}} = \frac{3\sqrt{6}}{8} \quad \sqrt{\frac{98}{63}} = \frac{7\sqrt{2} \cdot \sqrt{7}}{3\sqrt{7} \cdot \sqrt{7}} = \frac{2\sqrt{14}}{3 \cdot \cancel{7}} = \frac{\sqrt{14}}{3} \\ \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7} \quad \frac{\sqrt{6}}{2\sqrt{3}} = \frac{\sqrt{6} \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2} \\ \frac{5\sqrt{3}}{2\sqrt{5}} = \frac{5\sqrt{3} \cdot \sqrt{5}}{2 \cdot \sqrt{5} \cdot \sqrt{5}} = \frac{5\sqrt{15}}{10} = \frac{\sqrt{15}}{2} \quad \frac{3\sqrt{8}}{\sqrt{5}} = \frac{3 \cdot 2\sqrt{2}}{\sqrt{5} \cdot \sqrt{5}} = \frac{6\sqrt{2}}{5} \end{array}$$

5) Exercices de synthèseRéduis.

$$\sqrt{75} + \sqrt{50} = 5\sqrt{3} + 5\sqrt{2}$$

$$\sqrt{27} + \sqrt{3} = 3\sqrt{3} + \sqrt{3} = 4\sqrt{3}$$

$$\sqrt{27} \cdot \sqrt{3} = 3\sqrt{3} \cdot \sqrt{3} = 3 \cdot 3 = 9$$

$$\sqrt{8} + \sqrt{32} = 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$$

$$\sqrt{8} \cdot \sqrt{2} = 2\sqrt{2} \cdot \sqrt{2} = 2 \cdot 2 = 4$$

$$\left(\frac{2}{\sqrt{6}}\right)^2 = \frac{4}{6} = \frac{2}{3}$$

$$2\sqrt{45} + 3\sqrt{125} = 2 \cdot 3\sqrt{5} + 3 \cdot 5\sqrt{5} = 21\sqrt{5}$$

$$\sqrt{15} \cdot 2\sqrt{27} = \sqrt{5} \cdot \sqrt{3} \cdot 2 \cdot 3 \cdot \sqrt{3} = 6 \cdot 3 \cdot \sqrt{5} = 18\sqrt{5}$$

$$2\sqrt{18} + 3\sqrt{27} = 2 \cdot 3\sqrt{2} + 3 \cdot 3\sqrt{3} = 6\sqrt{2} + 9\sqrt{3}$$

$$3\sqrt{8} + 4\sqrt{32} = 3 \cdot 2\sqrt{2} + 4 \cdot 4\sqrt{2} = 22\sqrt{2}$$

$$5\sqrt{3} \cdot 3\sqrt{15} = 5 \cdot \sqrt{3} \cdot 3 \cdot \sqrt{3} \cdot \sqrt{5} = 15 \cdot 3 \cdot \sqrt{5} = 45\sqrt{5}$$

$$\frac{3\sqrt{2} \cdot 3\sqrt{6}}{\sqrt{27} \cdot \sqrt{3}} = \frac{9 \cdot \sqrt{12}}{3\sqrt{3} \cdot \sqrt{3}} = \frac{9 \cdot 2\sqrt{3}}{3 \cdot 3} = \frac{9 \cdot 2\sqrt{3}}{9} = 2\sqrt{3}$$

$$2\sqrt{8} + \sqrt{18} = 2 \cdot 2\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$$

$$2\sqrt{27} + \sqrt{45} = 2 \cdot 3\sqrt{3} + 3\sqrt{5} = 6\sqrt{3} + 3\sqrt{5}$$

$$(\sqrt{6} \cdot 2\sqrt{3})^2 = 6 \cdot 4 \cdot 3 = 72$$

$$\frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{27}} = \frac{\sqrt{6} \cdot \sqrt{3}}{3 \cdot \sqrt{3}} = \frac{\sqrt{6}}{3}$$